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Novel Approach to Renormalize the Electroweak Sector of the Standard Model

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Abstract

We discuss a novel approach to renormalize the Electroweak Sector of the Standard Model, in which $\sin^2\theta_{eff}^{lept}$, measured at the Z^0 peak, is identified with the basic renormalized electroweak mixing parameter. This approach shares the desirable convergence properties of the $\overline{MS}$ scheme and provides a framework for calculations that are strictly independent of the electroweak scale in finite orders of perturbation theory. We illustrate the method with updated precise calculations of $\sin^2\theta_{eff}^{lept}$ and M_W, as functions of M_H, and comment on the implications of these calculations for the Higgs boson search and the theoretical prediction of M_W.

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Current precise calculations in electroweak physics are based on a number of renormalization schemes. The on-shell [1, 2] and $\overline{MS}$ formulations [3, 4, 5, 6, 7], which are the most frequently employed, present a number of relative advantages and disadvantages. The on-shell approach is very "physical" in the sense that it identifies the renormalized parameters with observable and, therefore, scale-independent quantities, such as α, M_Z, and M_W. The $\overline{MS}$ calculations, on the other hand, have very desirable convergence properties. The reason is that they follow closely the structure of the unrenormalized theory and, in this way, avoid large finite corrections that frequently emerge in the renormalization process. They also involve parameters, such as $\hat{\alpha}(\mu)$ and $\sin^2 \hat{\theta}_W(\mu)$, which are inherently scale dependent, and play a crucial role in the analysis of grand unification.
On the other hand, since practical evaluations of important observables, such as $\sin^2 \theta_{eff}^{lept}$ and M_W, involve a truncation of the perturbative series, the use of the $\overline{MS}$ scheme necessarily leads to a residual dependence on the electroweak scale μ, thus generating a source of ambiguity.
Very recently, a framework has been proposed that retains the desirable convergence properties of the $\overline{MS}$ scheme and, at the same time, leads to calculations that are strictly independent of the electroweak scale in finite orders of perturbation theory [8]. In the present communication, we review some of the highlights of this approach and illustrate it with updated precise calculations of $\sin^2 \theta_{eff}^{lept}$ and M_W. We also comment on the implications of these calculations for the upper bound for the Higgs boson mass M_H, and the theoretical prediction of M_W based on the observed value of $\sin^2 \theta_{eff}^{lept}$.
An important feature of the proposed scheme is that $\sin^2 \theta_{eff}^{lept}$, obtained from the precise measurements of the various asymmetries at the Z^0 peak, is identified with the basic renormalized electroweak mixing parameter. It is related to the accurately known parameters α, G_μ, and M_Z by

$$s_{eff}^2 c_{eff}^2 = \frac{A^2}{M_Z^2\,(1-\Delta r_{eff})}\ , \tag{1}$$

where s_{eff}^2 is an abbreviation for $\sin^2 \theta_{eff}^{lept}$, $A^2 = \pi\alpha/\sqrt{2}G_\mu$ and Δr_{eff} is the relevant radiative correction. The one-loop approximation to Δr_{eff} has been recently used to discuss the mass scale of new physics in the Higgs-less scenario [9] and the evidence for electroweak bosonic corrections in the SM [10]. The main points in our strategy to evaluate Δr_{eff} beyond the one-loop approximation are:

i) Since current calculations of s^2_{eff} incorporate two-loop effects enhanced by powers $(M_t^2/M_Z^2)^n$ (with $n=1,2$), we first express Δr_{eff} in terms of a set of corrections $\Delta\hat{r}_W$, $\Delta\hat{\rho}$, $\Delta\hat{k}$ and $\hat{f}$, involving appropriate combinations of self-energies, vertex and box diagrams [3, 4, 5, 6, 7], for which the irreducible contributions of this order have been already evaluated [11, 12, 13].

ii) In order to ensure the absence of a residual electroweak-scale dependence, we use scale-independent couplings, such as e^2, s^2_{eff}, G_μ, M_Z^2, retain only two-loop effects enhanced by factors $(M_t^2/M_Z^2)^n$ $(n=1,2)$, since other two-loop corrections are not completely known, and employ a single definition of the electroweak mixing parameter, which we identify with $s^2_{eff}=1-c^2_{eff}$.

In particular, the $\overline{MS}$ parameter $\hat{s}^2(\mu)\equiv\sin^2\hat{\theta}_W(\mu)$ is expressed in terms of s^2_{eff} by means of the relation [7]

$$s^2_{eff}=\left[1+\frac{\hat{e}^2}{\hat{s}^2}\Delta\hat{k}\left(M_Z^2,\mu\right)\right]\hat{s}^2(\mu)\ , \tag{2}$$

where $\Delta\hat{k}(q^2,\mu)$ is the relevant electroweak form factor evaluated at the momentum transfer $q^2=M_Z^2$, and μ is the 't-Hooft scale.
In this way, we obtain the relation [8]

$$\begin{aligned}\Delta r_{eff} &= \Delta\hat{r}_W-\frac{e^2}{s^2_{eff}}\left[\Delta\hat{\rho}-\Delta\hat{k}\left(1-\frac{s^2_{eff}}{c^2_{eff}}\right)\right]\\ &- \frac{e^2}{s^2_{eff}}\,x_t\left[2\Delta\hat{\rho}-(\Delta\hat{\rho})_{lead}-\hat{f}+\Delta\hat{k}\frac{s^2_{eff}}{c^2_{eff}}\right]\ ,\end{aligned} \tag{3}$$

where $x_t=3G_\mu M_t^2/(8\sqrt{2}\pi^2)$ and $(\Delta\hat{\rho})_{lead}=(3/64\pi^2)\,M_t^2/M_W^2$ are the leading one-loop contributions to $(\hat{e}^2/\hat{s}^2)\,\Delta\hat{\rho}$ and $\Delta\hat{\rho}$, respectively, and we have neglected two-loop contributions not enhanced by powers of $(M_t^2/M_Z^2)^n$ $(n=1,2)$.
The corrections $\Delta\hat{r}_W$, $\Delta\hat{\rho}$, $\Delta\hat{k}$ and $\hat{f}$ depend also on $c^2=M_W^2/M_Z^2$, where c^2 is an abbreviation for the on-shell parameter $\cos^2\theta_W$ [1, 2]. In order to ensure the exact cancellation of scale dependences among various contributions, it is important to employ everywhere the same version of the electroweak mixing parameter which, as explained before, we identify with c^2_{eff}. To achieve this,

we proceed as follows:
a) M_W^2 is replaced everywhere by $c^2 M_Z^2$, b) in all two-loop contributions we substitute $c^2 \to c^2_{eff}$, since the difference is of third order, c) in the one-loop terms we perform a Taylor expansion, exemplified by

$$\Delta\hat{\rho}\left(c^2_{eff}, c^2\right) = \Delta\hat{\rho}\left(c^2_{eff}, c^2_{eff}\right) + \left.\frac{\partial\Delta\hat{\rho}}{\partial c^2}\right|_{c^2=c^2_{eff}} \left(c^2 - c^2_{eff}\right) + \cdots . \quad (4)$$

It suffices then to replace $c^2 - c^2_{eff}$ by its one-loop evaluation, expressed in terms of scale-independent couplings.
The corresponding expression for the calculation of M_W is given by [8]

$$M_W^2/M_Z^2 = c^2_{eff} \left\{ 1 - \frac{8\ G_\mu}{\sqrt{2}}\ M_Z^2\ c^2 \left[\Delta\hat{\rho} + \frac{s^2_{eff}}{c^2_{eff}} \Delta\hat{k}\,(1 - x_t) - \hat{f} x_t \right] \right\}^{-1} . \quad (5)$$

Again, c^2 is expressed in terms of c^2_{eff} in the manner explained above. An interesting feature of this formulation is that in Eqs.(1,2) the calculation of s^2_{eff} is carried out independently of M_W, while the s^2_{eff} results are employed in Eq.(5) to calculate M_W. For brevity, we refer to this approach as the Effective Scheme of Renormalization (EFF).
In order to illustrate our results, we employ $G_\mu = 1.16637 \times 10^{-5}\ GeV^{-2}$, $M_Z = 91.1875\ GeV$, $M_t = (174.3 \pm 5.1)\ GeV$, $\hat{\alpha}_s(M_Z) = 0.118 \pm 0.002$, QCD corrections based on the μ_t-parameterization [12, 14], and two recent determinations of $\Delta\alpha_h^{(5)}$: $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$ [15] and $\Delta\alpha_h^{(5)} = 0.02738 \pm 0.00020$ [16] (one of the "theory-driven" calculations). In Table 1 we present updated evaluations of s^2_{eff} and M_W, based on the Effective Scheme, as functions of M_H, using the central values of the input parameters and $\Delta\alpha_h^{(5)} = 0.02761$. In Table 2, we give the corresponding results for $\Delta\alpha_h^{(5)} = 0.02738$. Figs. 1 and 2 compare the scale dependence of the $\overline{MS}$ calculations of s^2_{eff} and M_W [12, 13] with the scale-independent Effective approach, for $M_H = 100\ GeV$ and $\Delta\alpha_h^{(5)} = 0.02761$. We note that for the value $\mu = M_Z$ selected in Refs.[12, 13], the two calculations are very close, with differences $\Delta s^2_{eff} = 0.8 \times 10^{-5}$ and $\Delta M_W = 0.6\ MeV$. On the other hand, we see that the Effective approach eliminates the ambiguity associated with the choice of the electroweak scale.
Inserting in Eqs.(1,2) the current experimental average $(s^2_{eff})_{exp} = 0.23156 \pm 0.00017$ and taking into account the uncertainties in the other input parameters, we find, for $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, the determination $M_H =$

140^{+97}_{-57} GeV and the 95% CL upper bound $M_H^{95} = 334$ GeV. On the other hand, inserting the current average $(M_W)_{exp} = 80.448 \pm 0.034$ GeV in Eq.(5), we obtain $M_H = 25^{+52}_{-25}$ GeV and $M_H^{95} = 131$ GeV. The corresponding values for $\Delta\alpha_h^{(5)} = 0.02738 \pm 0.00020$, are $M_H = 164^{+103}_{-63}$ GeV, $M_H^{95} = 364$ GeV from s^2_{eff}, and $M_H = 30^{+53}_{-30}$ GeV, $M_H^{95} = 139$ GeV from M_W. We see that the calculation based on the two values of $\Delta\alpha_h^{(5)}$ are rather close and that, at present, the experimental determination of M_W constrains M_H much more sharply than s^2_{eff}. In fact, the values of M_H we have obtained from M_W are well inside the 95% CL exclusion region from direct searches $M_H < 113$ GeV [17], and the corresponding upper bounds leave open but a small range above the exclusion limit! On the other hand, global analyses are less restrictive and lead, at present, to $M_H^{95} = 212$ GeV for $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, and to $M_H^{95} = 236$ GeV for $\Delta\alpha_h^{(5)} = 0.02738 \pm 0.00020$ [18].
Another interesting application is the use of $\ln(M_H/100\ GeV)$ derived from $(s^2_{eff})_{exp}$ and Eqs.(1,2), to predict M_W. In this way we find, with $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, the prediction $M_W = 80.366 \pm 0.026$ GeV, which differs from $(M_W)_{exp}$ by about 1.9σ! This is the first time over a period of several years that this calculation leads to a prediction of M_W that differs from $(M_W)_{exp}$ by more than 1σ and, in our opinion, it is an indication that the current fit to the SM is of lower quality than in the past. This is, of course, also strongly indicated by the low confidence level of the present experimental determination of s^2_{eff}.
In summary, we have outlined a novel framework to carry out precision calculations in the SM, in which s^2_{eff} plays the role of the basic electroweak mixing parameter. It shares the attractive convergence properties of the $\overline{MS}$ framework and, at the same time, eliminates the ambiguities associated with the choice of the electroweak scale. It may be particularly useful if s^2_{eff} can be measured with an error of $\pm 1 \times 10^{-5}$, as anticipated in the Tesla project.

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Table 1: Predicted values of M_W and s^2_{eff} in the EFF renormalization scheme for $M_t = 174.3\ GeV$, $\hat{\alpha}_s\left(M_Z^2\right) = 0.118$, $\Delta\alpha^{(5)}_{had} = 0.02761$, with QCD corrections based on the μ_t-parametrization.

$M_H\,[GeV]$	$M_w\,[GeV]$	$\sin^2\theta^{lept}_{eff}$
65	80.410	0.23117
100	80.386	0.23139
300	80.313	0.23196
600	80.260	0.23235
1000	80.220	0.23263

Table 2: As in Table 1, but with $\Delta\alpha^{(5)}_{had} = 0.02738$.

$M_H\,[GeV]$	$M_w\,[GeV]$	$\sin^2\theta^{lept}_{eff}$
65	80.414	0.23109
100	80.391	0.23130
300	80.317	0.23188
600	80.264	0.23227
1000	80.224	0.23255

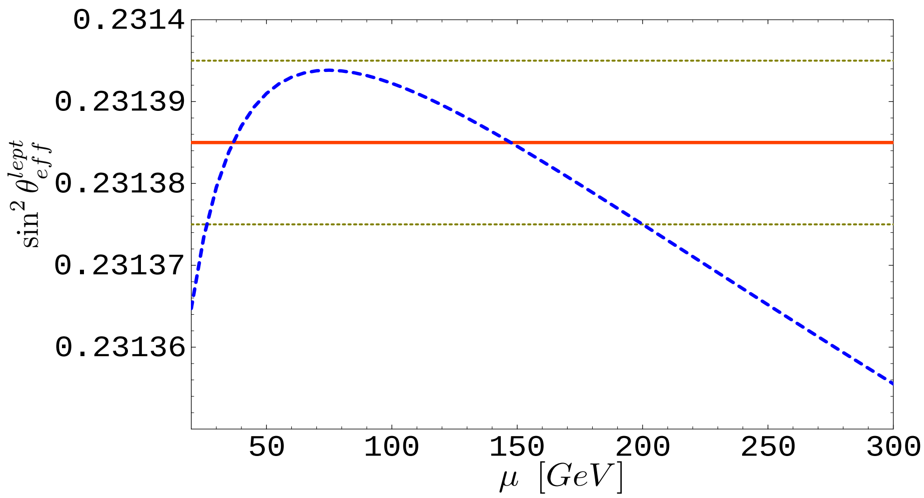

Figure 1: Scale dependence of s^2_{eff} in the $\overline{MS}$ (dashed line) and EFF (solid line) schemes for $M_H = 100\ GeV$ and the input parameters listed in Table 1. The light-dotted lines define a range of $\pm 1 \times 10^{-5}$ around the EFF result.

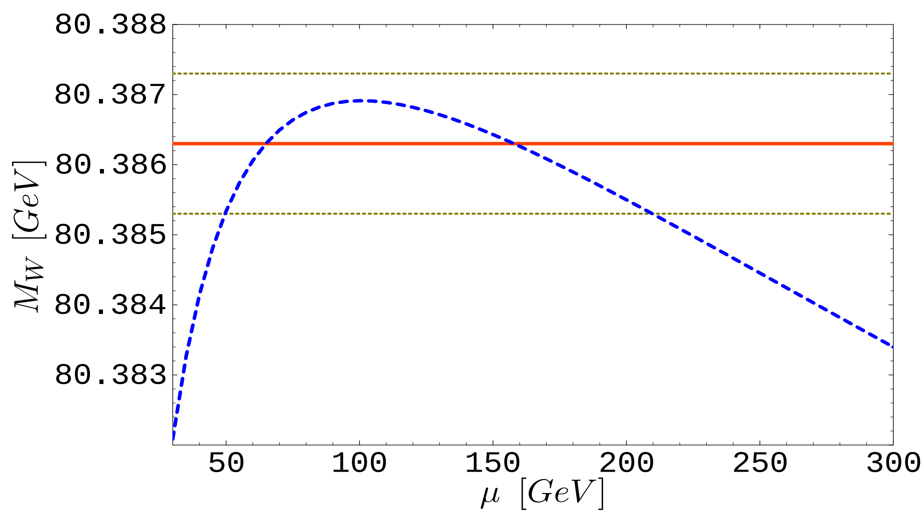

Figure 2: Scale dependence of M_W in the $\overline{MS}$ (dashed line) and EFF (solid line) schemes for $M_H = 100\ GeV$ and the input parameters listed in Table 1. The light-dotted lines define a range of $\pm 1\ MeV$ around the EFF result.